

from the operation of  $-a_2$ ,  $(111)[10\bar{1}]$  and  $b_1$ ,  $(11\bar{1})[011]$  were suppressed, a shear stress  $\tau_{yz}$  is generated to activate  $-d_1$  and  $d_2$  (in a direction to reduce  $d\epsilon_{yz}$ ).

### CONCLUSIONS

Depending on crystal orientation, the resistance to plane strain deformation can vary by a factor of two even in face-centered cubic material. This resistance arises mainly from the disposition of the slip systems with respect to the deformation geometry. Experiments on Permalloy single crystals confirmed the general validity of a Taylor and Bishop and Hill type analysis. However, we need to take into account such factors as lattice rotations, deformation banding, and relaxation of constraints in the deformation system. The tendencies for lattice rotations and for deformation banding are presumably related in an important way to details of dislocation interactions and would, therefore, depend on such factors as stacking fault energy, testing temperature and strain rates.

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### APPENDIX I

In the Bishop and Hill notation, the following designations are used to represent  $\{111\}\langle 110\rangle$  slip in f.c.c. metals:

Plane	(111)	( $\bar{1}\bar{1}\bar{1}$ )
Direction	01 $\bar{1}$ $\bar{1}01$ $\bar{1}\bar{1}0$	0 $\bar{1}\bar{1}$ $\bar{1}01$ $\bar{1}\bar{1}0$
Shear	$a_1$ $a_2$ $a_3$	$b_1$ $b_2$ $b_3$
Plane	( $\bar{1}\bar{1}\bar{1}$ )	( $\bar{1}\bar{1}\bar{1}$ )
Direction	01 $\bar{1}$ $\bar{1}01$ $\bar{1}\bar{1}0$	0 $\bar{1}\bar{1}$ $\bar{1}01$ $\bar{1}\bar{1}0$
Shear	$c_1$ $c_2$ $c_3$	$d_1$ $d_2$ $d_3$

The shear  $a_1$  represents slip on (111) plane and in the [01 $\bar{1}$ ] direction in the positive sense. The notation  $-a_1$  will be used for either ( $\bar{1}\bar{1}\bar{1}$ )[01 $\bar{1}$ ] or (111)[0 $\bar{1}\bar{1}$ ] slip, i.e., for shear in the negative sense.

Bishop and Hill have shown that the resolved shear stress on the twelve slip systems, multiplied by  $\sqrt{6}$ , are equal to:

$$\begin{aligned} \text{for } a_1, & A - G + H & a_2, & B + F - H \\ b_1, & A + G + H & b_2, & B - F - H \\ c_1, & A + G - H & c_2, & B + F + H \\ d_1, & A - G - H & d_2, & B - F + H \\ & & a_3, & C - F + G \\ & & b_3, & C + F - G \\ & & c_3, & C - F - G \\ & & d_3, & C + F + G, \end{aligned} \quad (\text{A1})$$

where

$$\begin{aligned} A &= \sigma_{22} - \sigma_{33}, & B &= \sigma_{33} - \sigma_{11}, & C &= \sigma_{11} - \sigma_{22} \\ F &= \sigma_{23}, & G &= \sigma_{31}, & H &= \sigma_{12}, \end{aligned}$$

all referred to the cubic axes.

Once the appropriate stress state from Table 1 is selected according to the maximum work principle, the values  $A$  to  $H$  are fixed. These values are then entered into equation (A1) to obtain the desired active slip systems. As an example, consider the (112)[ $\bar{1}\bar{1}\bar{1}$ ] deformation in the text (case 1). Stress state 21 from Table 1 is one of two states selected. For this state,  $A = -\frac{1}{2}$ ,  $B = \frac{1}{2}$ ,  $C = 0$ ,  $F = G = \frac{1}{2}$ ,  $H = 0$ , all multiplied by  $\sqrt{6} \tau$ . When these values are entered into equation (A1), the resolved shear stress for slip is reached only in systems  $-a_1$ ,  $a_2$ ,  $c_2$ ,  $-c_3$ ,  $-d_1$ , and  $d_3$ . Thus for  $-a_1$ ,  $(-A + G - H)/\sqrt{6} \tau = 1$ , and for  $b_1$ ,  $(A + G + H)/\sqrt{6} \tau = 0$  etc.

### APPENDIX II

#### Estimate of friction stress

A simple friction Hill analysis shows that under plane strain compression, the mean compressive stress  $\bar{\sigma}$  is equal to

$$\bar{\sigma} = \frac{\sigma_y h}{\mu L} (e^{\mu L/h} - 1), \quad (\text{A1})$$

where  $\sigma_y$  is the yield stress,  $h$  is the height,  $L$  is the length of the sample, and  $\mu$  is the coefficient of friction. The value of  $\mu$  is about 0.04 for teflon.<sup>(13)</sup> For the present samples, the initial dimensions are  $L \approx 0.4$  in.,  $h \approx 0.1$  in. Hence,  $\mu L/h \approx 0.16$  and  $\bar{\sigma} \approx 1.08\sigma_y$ . The friction contribution thus amounts to about 10% only. After 30% reduction, this contribution is calculated to be about 20%.

If the equation for sliding friction is used,<sup>(14)</sup>

$$\bar{\sigma}_f = \frac{\tau L}{3 h} \frac{1}{(1 - \rho)^{5/2}} \quad (\text{A2})$$

where  $\bar{\sigma}_f$  is the average friction stress,  $\tau$  is the yield stress in shear for teflon, and  $\rho$  is the fractional thickness reduction. For teflon the tensile yield stress is  $\sim 2000$  psi.<sup>(15)</sup> Hence  $\tau = 1000$  psi. Calculations of equation (A2) shows that  $\bar{\sigma}_f$  is about 3000 to 7000 psi in the range of 0 to 30% reduction. These values are about 10% of the flow stress of the Permalloy crystals.

The friction on the small side surface is complicated and depends among other factors on the spreading tendency of the crystal. This will contribute further to the scatter in the stress-strain curves.